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Lehmier Launch System

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LEHMIER LAUNCH SYSTEM

AKRONAUTS
ROCKET DESIGN TEAM

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Introduction

The Akronauts Rocket Design Team has been active at The University of Akron for three years now. After the first two years' success, the team decided to expand and compete in multiple competitions in 2017. The first of which, NASA Student Launch, is a distinguished competition with rigorous requirements for acceptance. The second competition is the Intercollegiate Rocket Engineering Competition (IREC) which the team has competed in twice before. This year, the Akronauts decided to build two rockets for IREC: one for the basic category and one for the advanced category. The latter would come to be the senior design rocket, named Zoom, built solely by the seniors on the team.

Building a rocket for the advanced category posed challenges that the members of the team have not faced previously. The target altitude, 30,000 feet, is three times the height of the past rockets. To reach this altitude, the team had to determine what the best method to do so was. After careful consideration, a single stage rocket was ultimately decided upon. To make a rocket weighing approximately 120 pounds reach 30,000 feet, a huge motor must be bought/built. The team's past rockets all utilized commercial motors, but after researching commercial motors and seeing the extreme cost of a motor the size needed, it was decided that the team needed to design and build one.

Amateur rocket motors are not as complex as the ones that take astronauts to the International Space Station, but they are still intricate. After all, it is rocket science. These motors are generally made up of six main components: motor case, nozzle, fuel, liner, forward closure, and aft closure. The motor case acts as a pressure vessel while the forward closure seals one end of the motor. The nozzle is held in place by the aft closure. To protect the motor case from the flowing gases produced by the fuel, a liner is inserted. Once the fuel is ignited, the gases produced flow through the nozzle, propelling the rocket upwards. **Figure 1** shows the locations of the main components of a rocket motor.

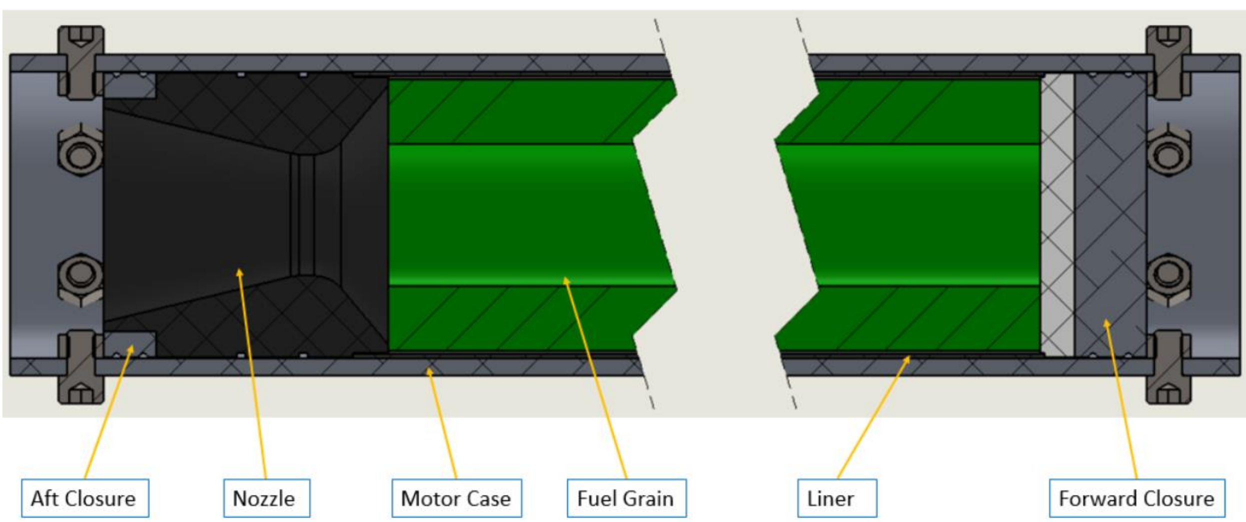


Figure 1 - Main Components of a Rocket Motor

Rocket Performance

There are a few important parameters to consider when it comes to a rocket motor's performance just as there are when considering the purchase of a new car. When looking at a new performance car, one may consider the vehicle's horsepower, torque, and fuel efficiency as frames of reference for what models they are interested in. Very similar to cars, rockets have certain measures of performance that are critical to consider when designing. The main ones are thrust and total impulse.

Thrust

Thrust is the force that propels that rocket into the air and is created from the motor. The rocket's motor burns fuel and accelerates the gases produced through the nozzle of the of the motor. From here, Newton's 3rd law of motion is utilized: for every action, there is an equal and opposite reaction. The gases being accelerated through the nozzle act downwards, therefore, forcing the rocket upwards. **Figure 2** shows a basic visual of thrust acting on a rocket.

$$T = \dot{m}V_e + (p_e + p_o)A_e$$

Total Impulse

Total Impulse is another measure of a rocket motor's performance that is critical to the design of a rocket. It is related to the thrust produced and the burn time of the motor. Specifically, it is defined as the integral of thrust over the burn time of the motor [1]:

$$I_T = \int_{t_0}^{t} F\, dt$$

The total impulse measures the momentum transferred to the rocket from the motor. In rocketry, there are different classes of motors for various ranges of total impulse [2].

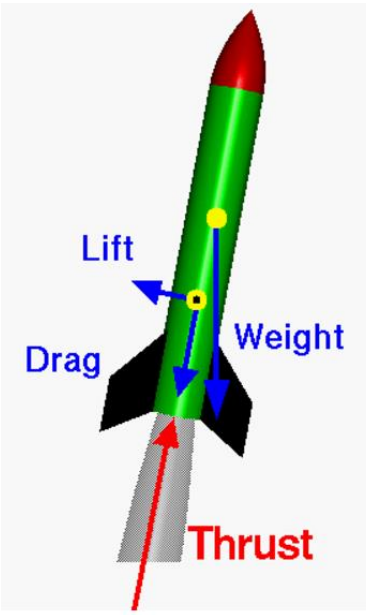


Figure 2 - Thrust Acting on a Rocket [3]

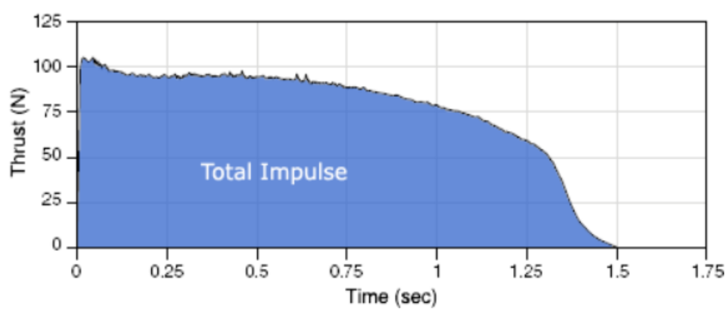


Figure 3 - Total Impulse [2]

Fuel

Many kinds of fuel are used in rocketry. High powered amateur rocketry usually utilizes either solid fuel, liquid fuel, or a hybrid of both. For Zoom, a solid fuel was chosen due to the simplicity. Using liquid or hybrid fuels adds the complexities of fuel injection and keeping the fuels cold enough to remain liquid. Solid rocket fuels are generally made of ammonium perchlorate and either aluminum or magnesium. Ammonium perchlorate and magnesium were chosen to be the fuel for Zoom.

Along with there being various kinds of fuel, there are many ways to cast the solid fuels. The geometry of the fuel grain alters the way the fuel burns and can have a large effect of the flight of the rocket. **Figure 4** on the right shows some of the different geometries that fuel can be cast in. The most common fuel geometry is a BATES (Ballistic and Test Evaluation System) Grain [5]. A BATES grain has a cylindrical core and burns from the top down and from the middle out. This creates a steady burn of the fuel throughout the burn. For Zoom, BATES grains were chosen.

Ammonium perchlorate and magnesium are not the only chemicals needed to make rocket fuel. They are just the most abundant. To create a good formula for the fuel, the team's mentor, Steve Eves, was contacted. With the help of his years of rocketry expertise, he was able to provide a complete formula for a fuel that would be suitable for the size of the motor being built. (Steve has been building rockets since he was a child and currently holds the world record for tallest and heaviest amateur rocket ever launched and recovered for his 1/10th scale Saturn V. His rocket is currently displayed at the U.S Space

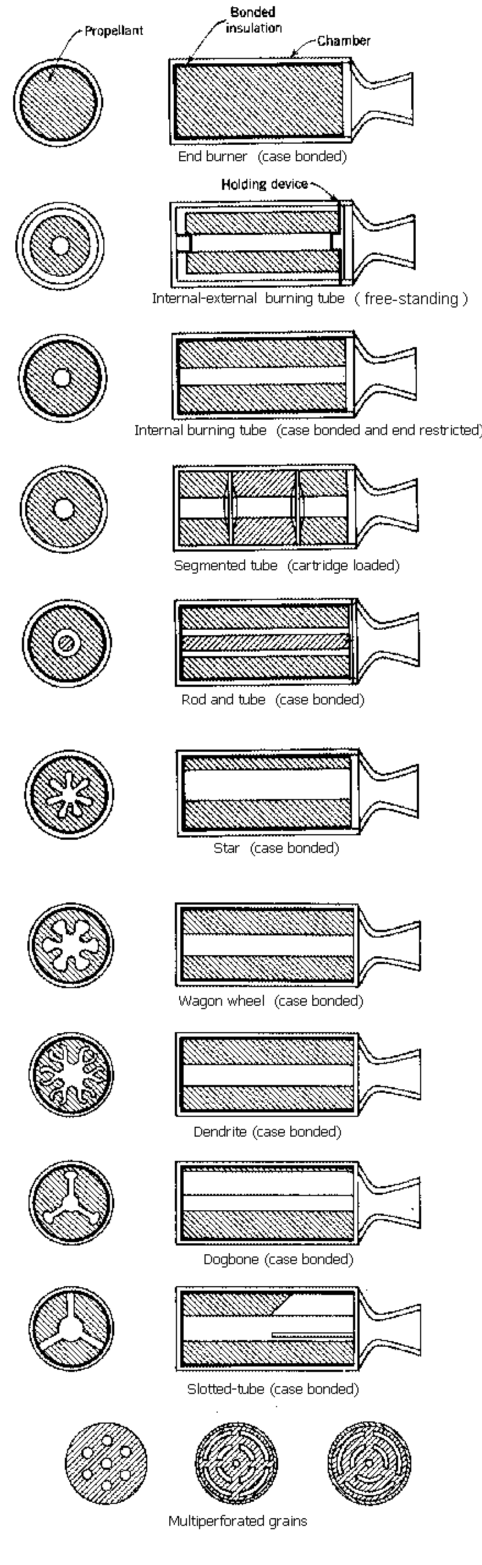


Figure 4 - Fuel Grain Design [4]

and Rocket center in Huntsville, Alabama.) The fuel formula can be seen in **Figure 5**.

Chemical Description	% of Batch
R45 HTLO	14%
Dioctyl Adipate, DOA	3%
Castor Oil	0.50%
Tepanol, HX-878	0.50%
Red Iron Oxide	0.10%
Zinc Dust	4%
Magnesium - 325 Mesh	6%
Ammonium Perchlorate 200um	70%
Modified MDI Isocyanate Curative	48.6116

Figure 5 - Fuel Formula

As seen in **Figure 5** there are nine chemicals included in this formula, all of which are widely used within the rocketry world. Each chemical was purchased online through Rocket Motor Components except for the ammonium perchlorate and magnesium. Steve has an abundance of these chemicals and was willing to sell some to the team for a discounted price.

The magnesium powder is the actual fuel and the ammonium perchlorate acts as the oxidizer. The other components have different purposes. The R45 HTLO (a type of hydroxyl-terminated polybutadiene) is the binder that holds the fuel together. The Modified MDI Isocyanate Curative is the curative that hardens the formula. The other components are added to change the burn rate of the fuel. The amount of curative was calculated using an excel macro based on the amount of each of the other compounds. The amount included in the formula is based on a 1500-gram batch of fuel. The proportions of each chemical are highly critical. A slight difference could cause a drastic change in burn rate, burn temperature, and chamber pressure. To ensure this doesn't happen, high precision in casting fuel must be achieved.

Casting Rocket Fuel

The actual casting process is not extremely difficult. The most important parts are using accurate scales to measure the individual chemicals and to repeat the process exactly the same every time. The complete process of casting fuel is seen in **Figure 6**. The mixing process can be done in an ordinary kitchen mixer. The mixer sees quite a large load depending on the fuel viscosity, because it needs to be running continually for over an hour. A fuel with a lower viscosity during mixture is generally more ideal. The mixer does not see as high of a loading, the air in the fuel is released more thoroughly under vacuum, and the fuel is easier to insert into casting tubes when it is softer.

Step	Description	Time (min)	Checklist
1	Mix liquids together (R45, DOA, Tepanol, Castor Oil)		
2	Mix	5	
3	Measure and Add Red Iron Oxide		
4	Mix	5	
5	Measure and add Magnesium		
6	Mix	5	
7	Measure and add Zinc Dust		
8	Mix	5	
9	Measure and add half of AP (doesn't have to be exactly half)		
10	Mix	5	
11	Add the rest of the AP (See 17 for other details)		
12	Mix	60	
13	While mixing, put masking tape around the casting tube		
14	Spray mold release on mandrel and casting caps		
15	Mix R45 and Curative together (18 grams and 4 grams respectively per grain)		
16	Apply mixture to inside of casting tube making sure to fill grooves		
17	After 45 minutes mixing, scrape down bowl		
18	Measure and add Curative		
19	Mix	10	
20	Scrape down bowl and mixer		
21	Mix	5	
22	Scrape down bowl and mixer		
23	Vacuum	5	
24	Prepare to cast fuel on mandrel while under vacuum		
25	Roll and tamp until grain is finished		
26	Add top casting cap		
27	Repeat for next grain		

Figure 6 - Fuel Casting Procedure

As seen in **Figure 6**, the entire process of casting fuel takes approximately 2 hours per batch. Fuel was cast alongside of Steve Eves to ensure that it was done properly. The first dozen steps in casting fuel are relatively self-explanatory. After that, it may get a little confusing. Solid rocket fuel is cast in casting tubes that fit inside the motor case's liner. These casting tubes need to be lined with a mixture of the same binder and curative that is used in the fuel. This prevents any defects in the casting tube from causing issues during the burn. Based on a BATES grain, there needs to be a cylindrical hole in the center of the grain. Instead of casting a solid grain and drilling out a hole, in this case a 2-inch hole, and wasting all of that fuel, the fuel is cast around a mandrel. The mandrel is centered in the casting tube using casting caps that were 3D printed to the exact size needed (see **Figure 7**). It is best to prepare the casting tubes during the long mixture of the fuel, because once the curative is added, the fuel needs to be placed inside the tubes rather quickly. Once the curative has been added and the mixing process is complete, the fuel needs to go under vacuum to take all the air out of the fuel. To do this, a cheap vacuum pump was purchased that was attached to a piece of polycarbonate with a seal the same size of the mixing bowl. When the vacuum is applied to the fuel, it starts to grow considerably. This is due to the air being pulled out of the fuel. (It's shocking how much the volume increases while the air is being extracted.) Once the fuel has been vacuumed for 5-minutes, it can be put into the casting tubes. The fuel needs to be tamped down to make sure there are no bubbles within the grain. A bubble could cause catastrophic failure. Once the casting tube is completely full, apply the top casting cap, and that grain is complete. Zoom's motor requires 11 grains.

Figure 7 - Assembly of Casting Tubes, Casting Caps, and Mandrel

BurnSim

One of the most useful programs in rocketry is BurnSim. BurnSim is, as you may guess, is a fuel burn simulation. Within this program, fuel characteristics can be entered and modified to see what kind of chamber pressure, thrust, mass flow rate, impulse, and many other useful values are for your type of fuel and amount of fuel. Using BurnSim to model the combustion of the rocket fuel being used was one of the first steps in determining the size of the motor needed.

Once the formula for the fuel was known, the values were entered into BurnSim. The values in **Figure 8** were provided from Steve along with the fuel formula.

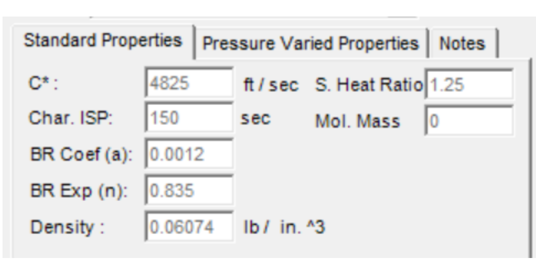


Figure 8 - BurnSim Fuel Characteristics

Using this information, the motor size needed to be determined. As a general reference, to get a rocket to fly 30,000 feet, the motor needs to be somewhere around 30,000 N-s of total impulse. Using that as a frame of reference, different lengths and geometries of fuel were tried. As a starting point, a 98mm (3.875-inches) motor case was chosen. This was quickly ruled out when it was determined that a 30,000 N-s motor would need to be nearly 10-feet tall. From there, the motor case was increased in size to 114mm (4.5-inches). Using this size motor, the fuel grains would be 3.8-inch in diameter. **Figure 9** shows the grain geometry input in BurnSim. The sizing shown is the grain sized being used in Zoom's motor.

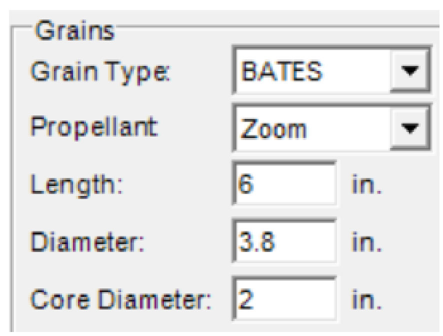


Figure 9 - BurnSim Grain Geometry

Fuel grain geometry is not the only thing BurnSim is useful for. It also has a nozzle geometry generator. The rocket nozzle is one of the most important components in the rocket motor (see nozzle section below). A slight change in the geometry can alter the performance drastically. Luckily, BurnSim can calculate rough dimensions for the nozzle. The nozzle throat can be entered and the best exit diameter for the nozzle is calculated. Unfortunately, the exit diameter is limited to the diameter of the motor and the aft closure. After trial and error of nozzle throat diameter, 1.7-inches was determined to be the best. This throat diameter was combined with an exit diameter of 3-inches. With these values determined, the motor size and performance was complete. **Figure 10** shows the final BurnSim simulation that would be used.

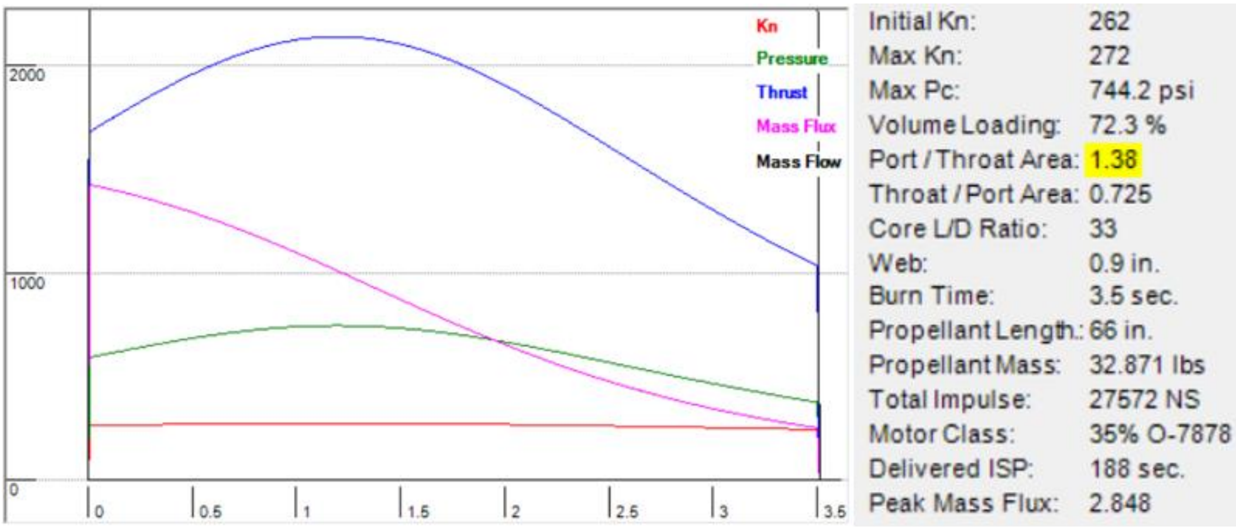


Figure 10 - BurnSim Simulation Results

Motor Case and Hardware

The next part of designing the rocket is choosing a motor case. The motor case acts as the pressure chamber that houses the fuel for the rocket. There are some basic sizes of motor casings that are used in the industry. Generally, motors are one of the following sizes: 38mm, 54mm, 75mm, 98mm, 114mm, and 152mm. As stated before, the motor for Zoom is 114mm. The length of the case must be long enough to house all the fuel (66-inches long) and must be a large enough diameter to contain the liner and fuel grains. With the grains being 3.8-inches in diameter, a liner with that approximate size needed to be purchased. Rocket motor liners are generally made of phenolic tubing. Public Missiles sells phenolic tubing that are used for rocket airframes, but they can also be used as liners and casting tubes. A properly sized tube was purchased with an outside diameter of 4-inches. This outside diameter needs to fit rather snuggly inside the case.

Motor Case Material Selection

The motor case could be made from any metal tube/pipe. The material needs to have certain structural requirements, needs to be machinable, and cannot be too pricey. To determine the material to use, some skills learned in Concepts of Design were utilized. The objective tree in **Figure 11** was made to show which factors were most important when choosing a material.

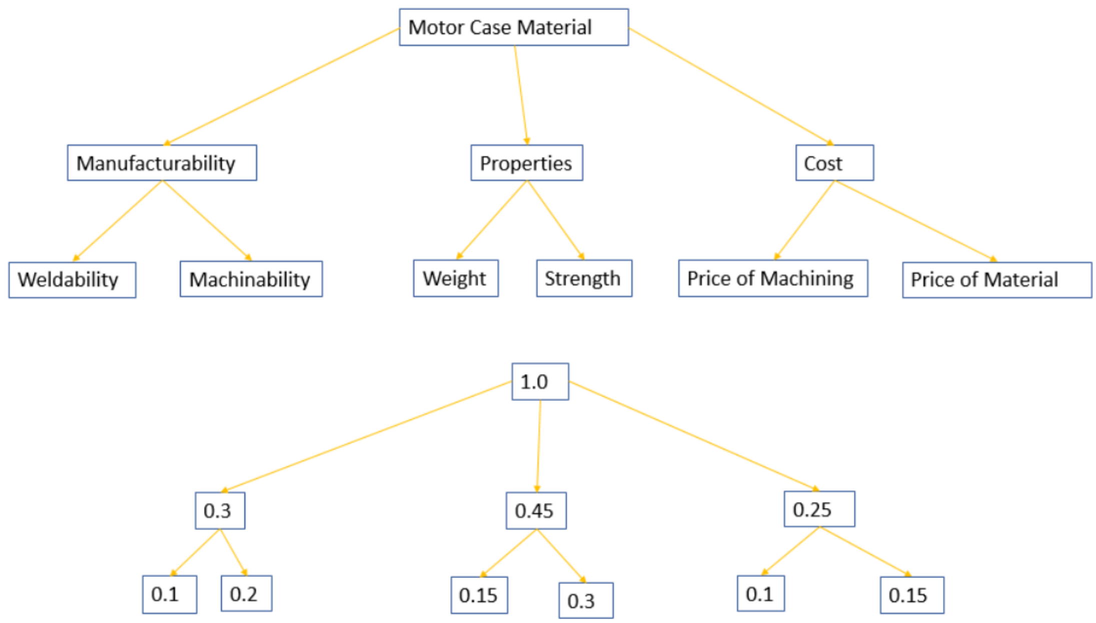


Figure 11 - Objective Tree to Determine Motor Case Material

The objective tree shows how the needs of the material were rated. The possible materials to be used for the motor case were closely examined. The weighted decision matrix in **Table 1** was used to choose between the possibilities. The value used in the weighted decision matrix were determined using an online material comparison website [6].

Criteria	Weight	6061-T6		304 Stainless		6063 Aluminum	
		Value	Rating	Value	Rating	Value	Rating
Weldability	0.1	7	0.7	8	0.8	7	0.7
Machinability	0.2	6	1.2	6	1.2	6	1.2
Weight	0.15	8	1.2	4	0.6	8	1.2
Strength	0.3	7	2.1	8	2.4	4	1.2
Price of Machining	0.1	7	0.7	8	0.8	7	0.7
Price of Material	0.15	9	1.35	6	0.9	9	1.35
		Total	**7.25**	**Total**	**6.7**	**Total**	**6.35**

Table 1 - Motor Casing Material Weighted Decision Matrix

As seen above, the possible materials were narrowed down to two aluminum alloys and one stainless steel. The strength of the material was the most crucial factor, but the strongest material was not chosen due it's lesser rating in the other criteria. 6061-T6 aluminum was ultimately chosen as the winner.

Motor Case Strength Analysis

Motor cases need to be strong enough to withstand the internal pressure produced from the fuel. To ensure that the chosen case is strong enough, a stress analysis was performed. Ideally, the motor case would have a wall thickness of 1/8-inch, but difficulties in finding a tube to match these dimensions led to the purchase of a 6062-T6 schedule 40 pipe. This pipe has a wall thickness of 0.24-inches and an outer diameter of 4.5-inches. The increase in wall thickness caused the motor to have extra weight, but increased the safety factor of the case.

Since the motor case ended up being thicker, the stress analysis was no longer able to be done using a thin-walled approximation. To determine the stresses in a thick-walled pressure vessel, the following equations were used [7]:

$$\sigma_a = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2}$$

$$\sigma_c = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} - \frac{r_i^2 r_o^2 (p_o - p_i)}{r^2 (r_o^2 - r_i^2)}$$

$$\sigma_r = \frac{p_i r_i^2 - p_o r_o^2}{r_o^2 - r_i^2} + \frac{r_i^2 r_o^2 (p_o - p_i)}{r^2 (r_o^2 - r_i^2)}$$

Where:

Variables used in Stress Calculation	
σ_a	Stress in the axial direction
σ_c	Stress in the circumferential direction
σ_r	Stress in the radial direction
p_i	Internal pressure in the tube
p_o	External pressure in the tube
r_o	Outer radius of tube
r_i	Inner radius of tube
r	Radius to a point in tube

Table 2 – Nomenclature for Thick Walled Stress Equations

As shown previously, BurnSim can predict what the chamber pressure is going to be in the motor case during the burn. The stress calculations were done using an internal pressure of 744.2 psi. The maximum stress will occur when $r = r_i$, so that value will be used in the following calculations [7]. To get a visualization of how the stresses act on the motor case see **Figure 12**.

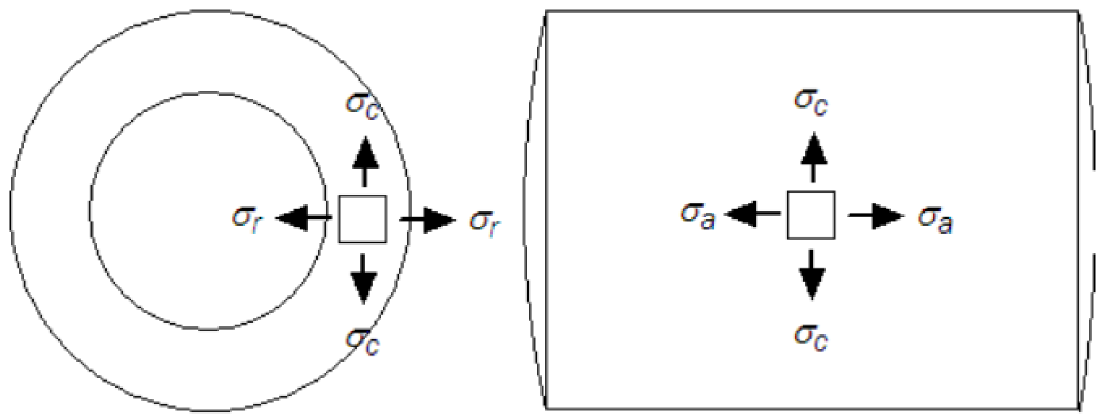


Figure 12 -Stresses on Thick-Walled Pressure Chamber [7]

The calculations for stresses were done using the MATLAB code found in **Appendix 1**. This code generates the three stresses acting on the motor case, the maximum internal pressure that can be applied to the motor case before it fails, and the factor of safety for the predicted internal pressure compared to the maximum internal pressure. The results are as follows:

```
The Axial Stress is 2911.13 psi.

The Circumferential Stress is 6566.46 psi.

The Radial Stress is -744.20 psi.

When the internal pressure reaches 4624.0 psi, the stress exceeds the
tensile yield strength (40000.0 psi) and is likely to fail.

The factor of safety of the motor casing is 6.2.
```

The results show that the motor case has a factor of safety greater than 6, proving that the motor is very safe to operate at these conditions. The maximum internal pressure this case withstand is 4624 psi. Knowing this, the internal pressure could be altered by changing the geometry of the nozzle to try to create a higher total impulse and thrust. For Zoom, it will remain the same for safety purposes.

Aft Closure Design Selection

The aft closure of the rocket motor is designed to retain the rocket nozzle while providing a tight seal to assure the gases are only flowing through the nozzle. There are several types of aft closures that can be used when designing a nozzle. When deciding on how to retain the nozzle, manufacturability was used as the biggest restraint. Because the motor case was so large, some of the aft closure types would be ruled out due to machining expenses/capabilities. To compare the multiple type being considered, another objective tree and weighted decision matrix were formulated. **Table 3** shows the main designs being considered.

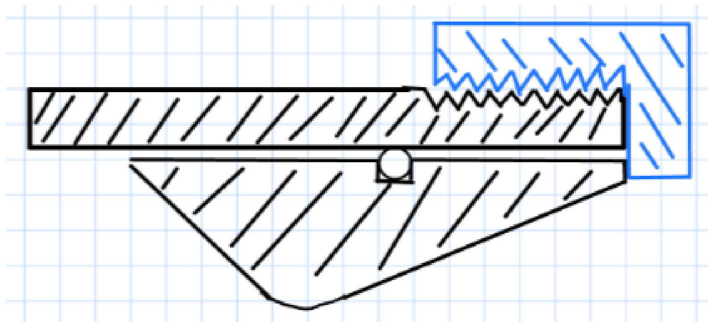
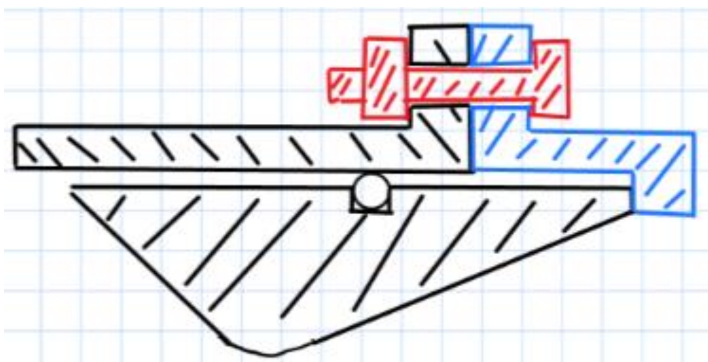
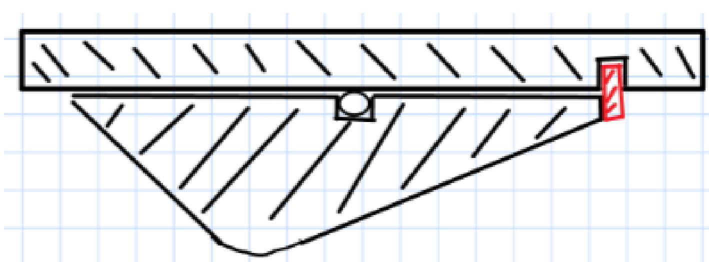
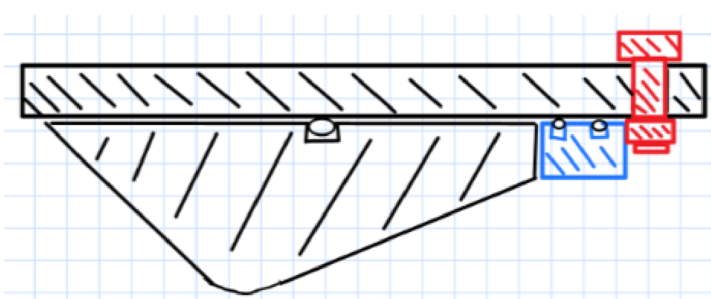
Aft Closure Designs	
	Threaded Retention
	Bolted Retention
	Snap Ring Retention
	Bolted Bulkhead Retention

Table 3 – Possible Aft End Closures/Nozzle Retention

Table 3 shows visuals of how the four-main aft closure designs retain the nozzle in the motor. These four designs were carefully analyzed to determine which design would be best. The objective tree in **Figure 13** and the weighted decision matrix in **Table 4** were used to determine which of the four designs would be used.

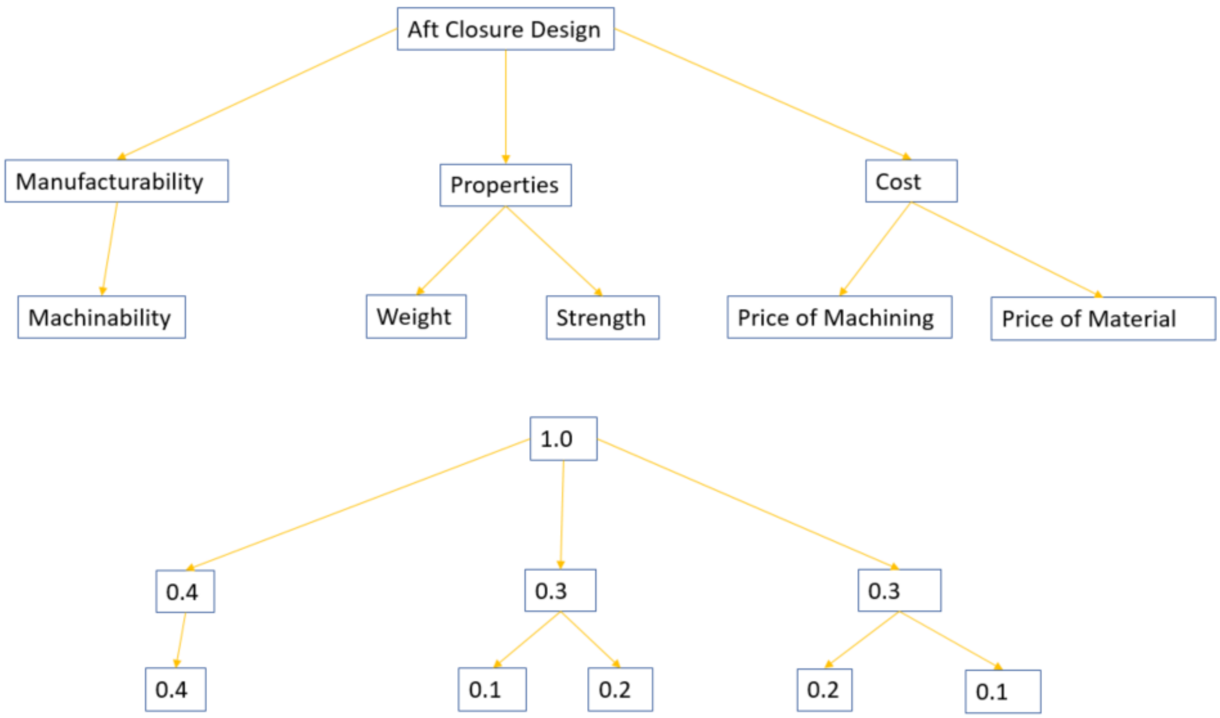


Figure 13 - Objective Tree to Determine Aft Closure Design

Criteria	Weight	Threaded		Bolted		Snap Ring		Bolted Bulkhead	
		Value	Rating	Value	Rating	Value	Rating	Value	Rating
Machinability	0.4	5	2	4	1.6	8	3.2	9	3.6
Weight	0.1	6	0.6	7	0.7	9	0.9	7	0.7
Strength	0.2	7	1.4	8	1.6	6	1.2	8	1.6
Price of Machining	0.2	5	1	5	1	8	1.6	8	1.6
Price of Material	0.1	6	0.6	6	0.6	9	0.9	7	0.7
		Total	5.6	Total	5.5	Total	7.8	Total	8.2

Table 4 – Aft Closure Design Weighted Decision Matrix

As mentioned before, when determining a design for the aft end closure, the most important part was making it simple to machine. As shown in the weighted decision matrix, the machinability was the difference maker in using the bolted bulkhead retention design instead of the snap ring retention. The difficulties in threading the motor case were determined early as this was the initial design idea. After calling numerous machine shops and companies that may be able to thread a pipe this size, it was determined that this was not the best design. Most machine shops were not able to thread a pipe that size, and the ones that were did not take on small, low paying jobs. Not only would the case have to be threaded, the aft closure would have to be threaded as well. The bolted retention design would need an extraordinary amount of machining. The motor case would have needed to be ordered with a larger wall thickness and then turned on a lathe to create the lip on the aft end. The snap ring was almost the winning design in the weighted decision matrix. This design would have added the least amount of weight and cost the least, but the strength was low. Machining the lip into the motor case causes a high stress area in the snap ring groove.

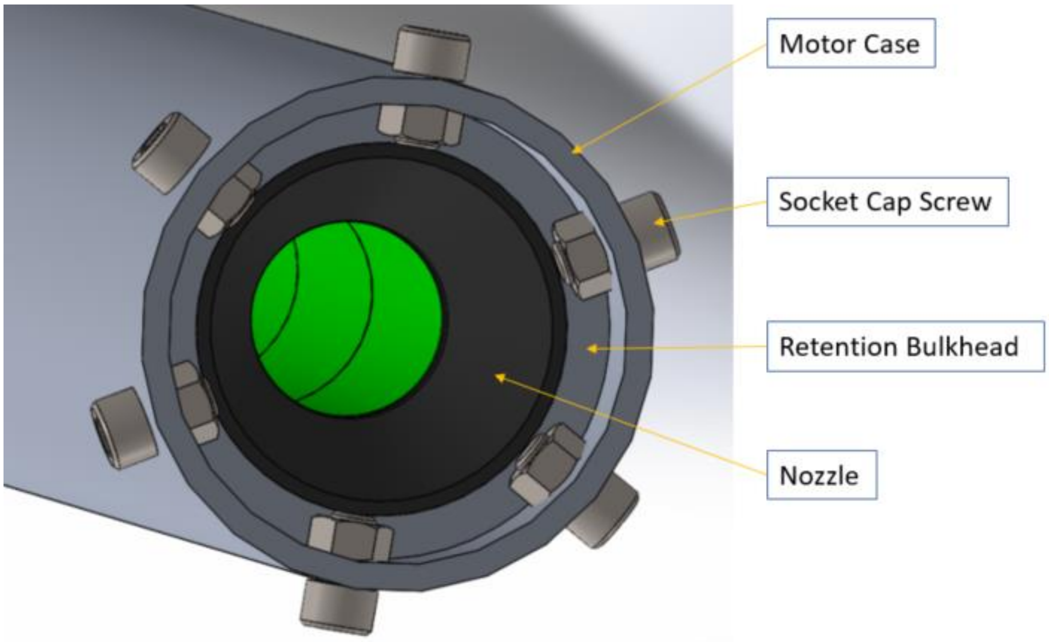


Figure 14 - Bolted Bulkhead Retention

The retention bulkhead includes two grooves for o-rings that will seal the end of the motor. The six socket cap screws will be made of class 12.9 alloy steel and use the flat surfaces of the nuts to hold the bulkhead in place. The minimum yield strength for class 12.9 alloy steel bolts is 1100 MPa (160,000 psi) [8]. Socket cap screws were chosen to provide an even surface for the Akronaut's aerostructure team to hold the motor in the rocket using thrust rings. A MATLAB code (see **Appendix 2**) was created to determine the factor of safety of using the six bolts. The stresses determined earlier in the motor case analysis were used. The safety factors of shear stress on the screws, bearing stress on the screws, and bearing stress on the motor hardware were found. The results were as follows:

```
Safety Factor for Shear of using 6 screws: 18.6060

Safety Factor for bearing stress of using 6 screws: 30.7680

Safety Factor for bearing stress on hardware: 6.7305
```

As seen above from the ridiculously high safety factors, 6 screws seem to be overkill. In fact, 1 of these socket cap screws would be strong enough, but obviously 1 screw would not be able to hold the bulkhead on the motor. Less screws could be used, but having the screws placed equally at $60°$ distributed the forces along the bulkhead and assures the nozzle does not shift during the flight.

Forward Closure Design Selection

The process in determining the forward closure was very similar to the process in determining the aft closure. The three design possibilities that were considered are shown in **Table 5**.

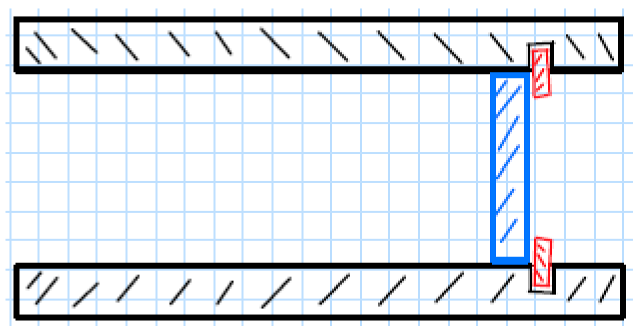
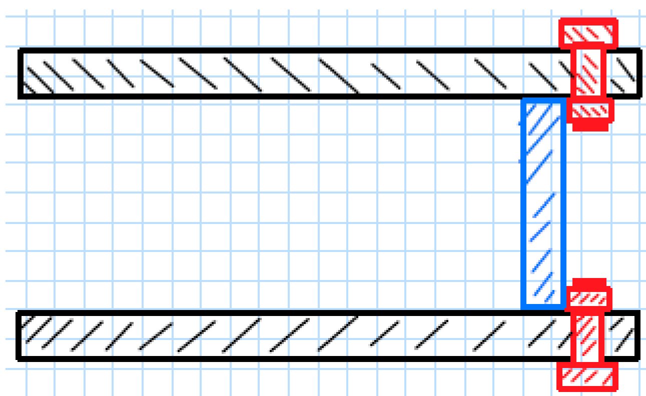
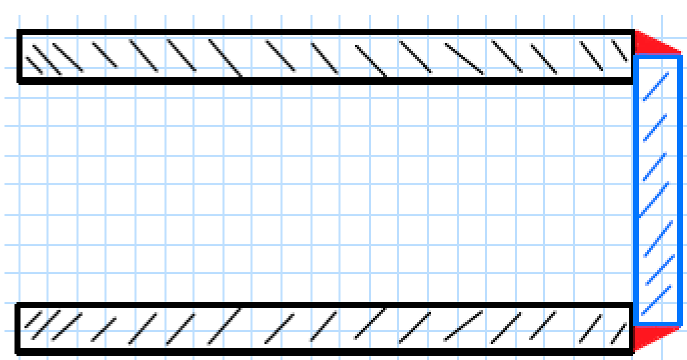
Forward Closure Designs	
	Snap Ring Closure
	Bolted Bulkhead Closure
	Welded Closure

Table 5 – Possible Forward End Closures

Another objective tree and weighted decision matrix were formed and it was determined that the best design for the forward closure would be the welded closure. This simple design would be strong, light weight, and easily manufactured. One factor that was not considered was reusability of the motor. If one end is permanently sealed, it makes clearing the motor case of the liner nearly impossible. The liner expands as it burns. It is already a very snug fit within the motor case and usually needs scraped out to reuse the motor case. With this design scrapped, the forward closure was chosen to be the same as the aft closure: the bolted bulkhead closure.

Using the same design for the aft and forward closures adds some simplicity to the motor. The same calculations can be used for each. An added benefit of using the bolted bulkhead design for the forward closure is the ability to attach the airframe to the motor. By putting the screws through the airframe and into the motor casing the same way as aft closure, it adds structural integrity to the motor bay of the rocket. (The motor bay is part of the aerostructure design of the rocket being done by another student.)

Nozzle

Rockets use a nozzle to accelerate hot gases produced from the motor to high speeds creating thrust. In the thrust equation provided earlier, exit velocity, V_e, is in the first term in the equation. This term is critical in producing a large amount of thrust. To get the exit velocity above Mach 1, the speed of sound, a converging-diverging nozzle must be used.

A converging-diverging nozzle, seen in **Figure 15**, seems a little redundant at first. When first learning about nozzle and diffusers, one learns that a nozzle is needed to increase the flow velocity and a diffuser is used to decrease the flow velocity. Well a converging-diverging nozzle is basically a nozzle with a diffuser attached to the back of it. Why would you ever want to slow the flow after you just accelerated it? In fact, this must be done to increase the speed even farther.

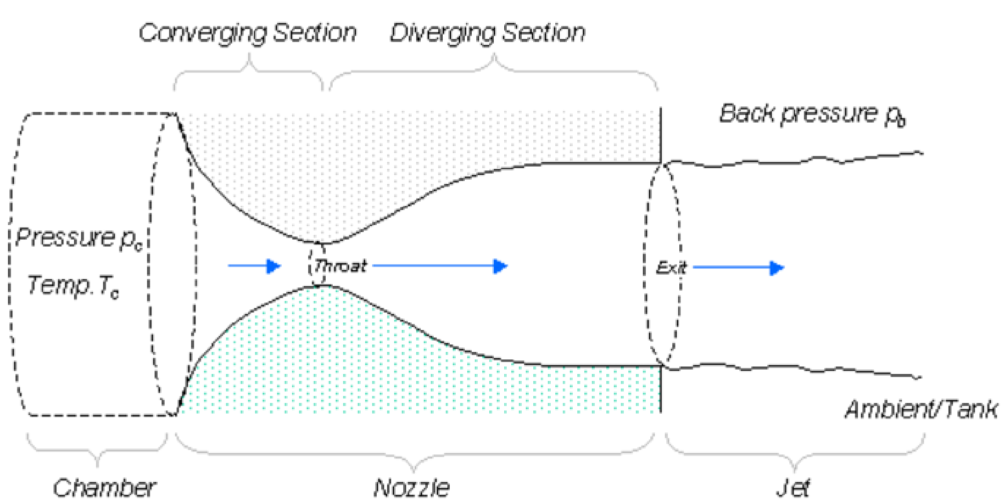


Figure 15 - Converging-Diverging Nozzle [10]

Looking at Figure 15, the various parts of a converging-diverging nozzle are clearly defined. The first half of the nozzle is the converging section and the second have is the diverging section. Where these two meet is called the nozzle throat. The throat acts as a choke and sets the mass flow rate through the motor. At the throat, the flow is at sonic velocity, Mach 1. Immediately past the throat, the nozzle diverges and this causes the flow to isentropically expand to a supersonic flow velocity [10]. The following equation for isentropic flow in a nozzle helps explain [11]:

$$\frac{dV}{V}(M^2 - 1) = \frac{dA}{A}$$

Where:

Variables	Name
dV	Change in Velocity
V	Velocity
M	Mach Number
dA	Change in Area
A	Area

Table 6 – Variables Described in Nozzle Equation

Looking at this equation from the standpoint of a regular converging nozzle:

- For subsonic flow (M<1) in a converging nozzle (dA<0) the velocity increases (dV>0)
- For supersonic flow (M>1) in a converging nozzle (dA<0) the velocity decreases (dV<0)

Looking at this equation from the standpoint of a regular diverging diffuser:

- For subsonic flow (M<1) in a diverging diffuser (dA>0) the velocity decreases (dV<0)
- For supersonic flow (M>1) in a diverging diffuser (dA>0) the velocity increases (dV>0)

So basically, a normal converging nozzle can only accelerate the flow to a certain point (Mach 1) before it no longer increases the velocity of flow and a diffuser only decelerates the flow when it is subsonic. Therefore, putting the two features together and accelerating the flow using a nozzle to Mach 1 and then immediately expanding into a diffuser, the flow velocity can be increased much higher than Mach 1. All of this directly relates back to the thrust equation. The exit velocity of gas through the rocket motor can now be higher and increase thrust produced.

To determine the thrust produced by the rocket motor, the following isentropic flow equations must be solved [9]:

Mass Flow Rate: $$\dot{m} = \frac{A^* p_t}{\sqrt{T_t}} \sqrt{\frac{\gamma}{R}} \left(\frac{\gamma+1}{2}\right)^{\frac{-\gamma+1}{2(\gamma-1)}}$$

Exit Mach: $$\frac{A_e}{A^*} = \left(\frac{\gamma+1}{2}\right)^{\frac{-\gamma+1}{2(\gamma-1)}} \frac{\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e}$$

Exit Temperature: $\frac{T_e}{T_t} = (1 + \frac{\gamma - 1}{2} M_e^2)^{-1}$

Exit Pressure: $\frac{p_e}{p_t} = (1 + \frac{\gamma - 1}{2} M_e^2)^{\frac{-\gamma}{\gamma - 1}}$

Exit Velocity: $V_e = M_e \sqrt{\gamma R T_e}$

Thrust: $T = \dot{m} V_e + (p_e + p_o) A_e$

Where:

Variables	Name
γ	Specific Heat Ratio
R	Gas Constant
A^*	Nozzle Throat Area
A_e	Nozzle Exit Area
M_e	Exit Mach
$\dot{m}$	Mass Flow Rate
p_e	Exit Pressure
p_t	Total Pressure
T_e	Exit Temperature
T_t	Total Temperature
V_e	Exit Velocity

Table 7 – Variables Described in Isentropic Flow Equations

According to the BurnSim simulation discussed previously, the maximum thrust was determined to be 2152.804 psi with an average of 1780.028 psi. The maximum mass flow rate was determined to be 11.26 lbs/sec with an average of 9.39 lbs/sec.

Model and Simulation

Once the simulation on BurnSim was completed and the motor hardware design was finalized, a 3D model of the nozzle could be made. BurnSim provides the throat diameter and exit diameter needed and the hardware design provides the way that the nozzle will be retained within the motor itself. Solid propellant rocket nozzles are made from graphite. The graphite is easily machined and can withstand multiple uses. The graphite available for purchase was a limiting factor in the nozzle design. To purchase a graphite slug at the diameter and length needed, it was quite expensive. Luckily, some cheap slugs of graphite were found, but they were only 4 inches in length. This shorter design of a nozzle turned out to be less efficient. The 3D model of the nozzle can be seen in **Figure 16**.

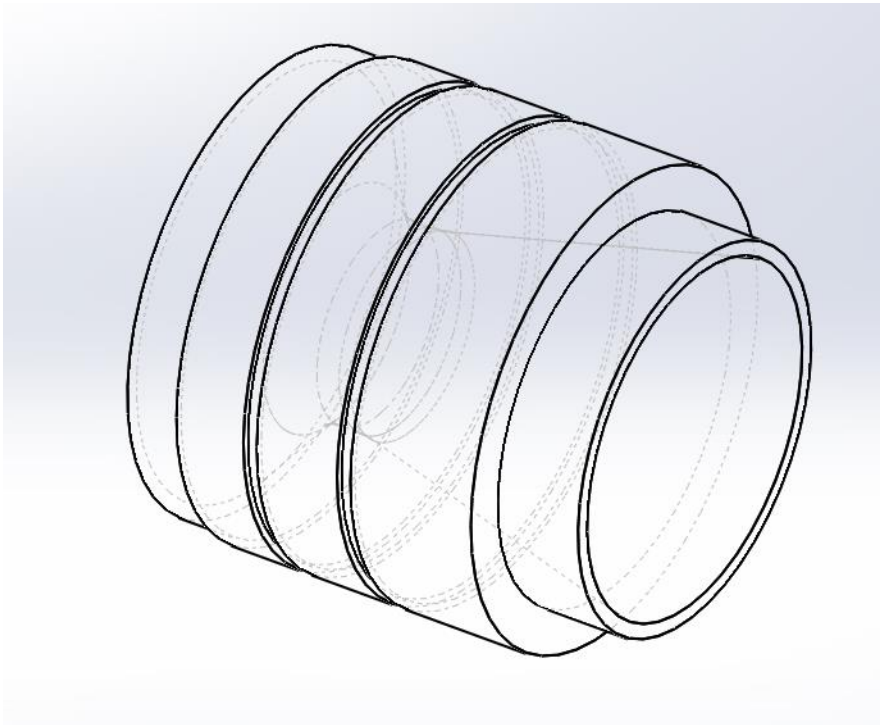


Figure 16 - 3D Model of Nozzle Designed in Solidworks

Once the model was completed, a computational fluid dynamics (CFD) simulation was completed using ANSYS Fluent. Using this simulation, the velocities and pressures within the nozzle could be determined and visualized. **Figure 17** shows the velocity contour in the nozzle and **Figure 18** shows a progression of fluid flow through the nozzle.

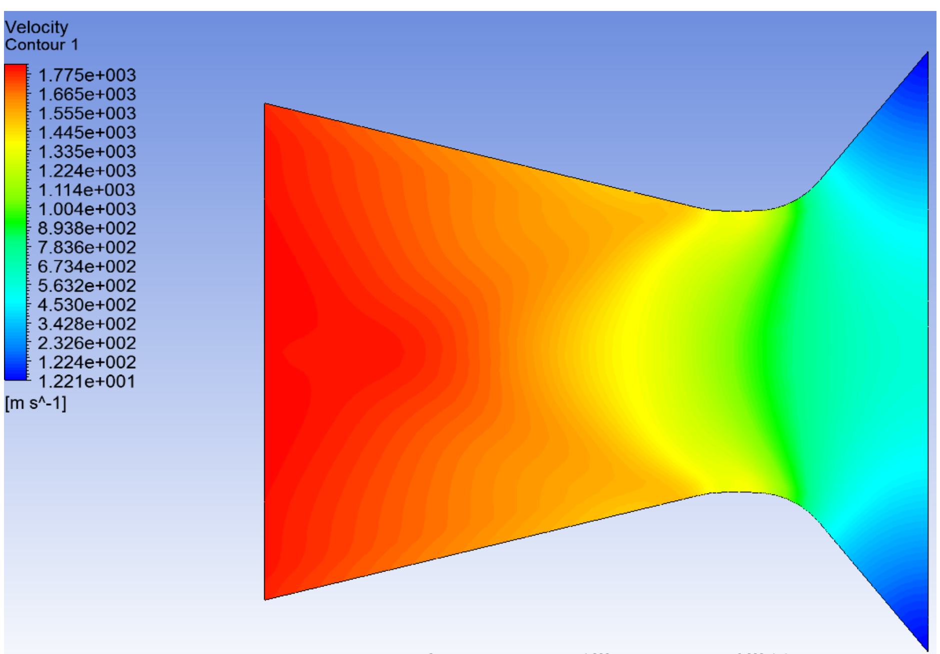


Figure 17 - Nozzle Velocity Contour Through Nozzle

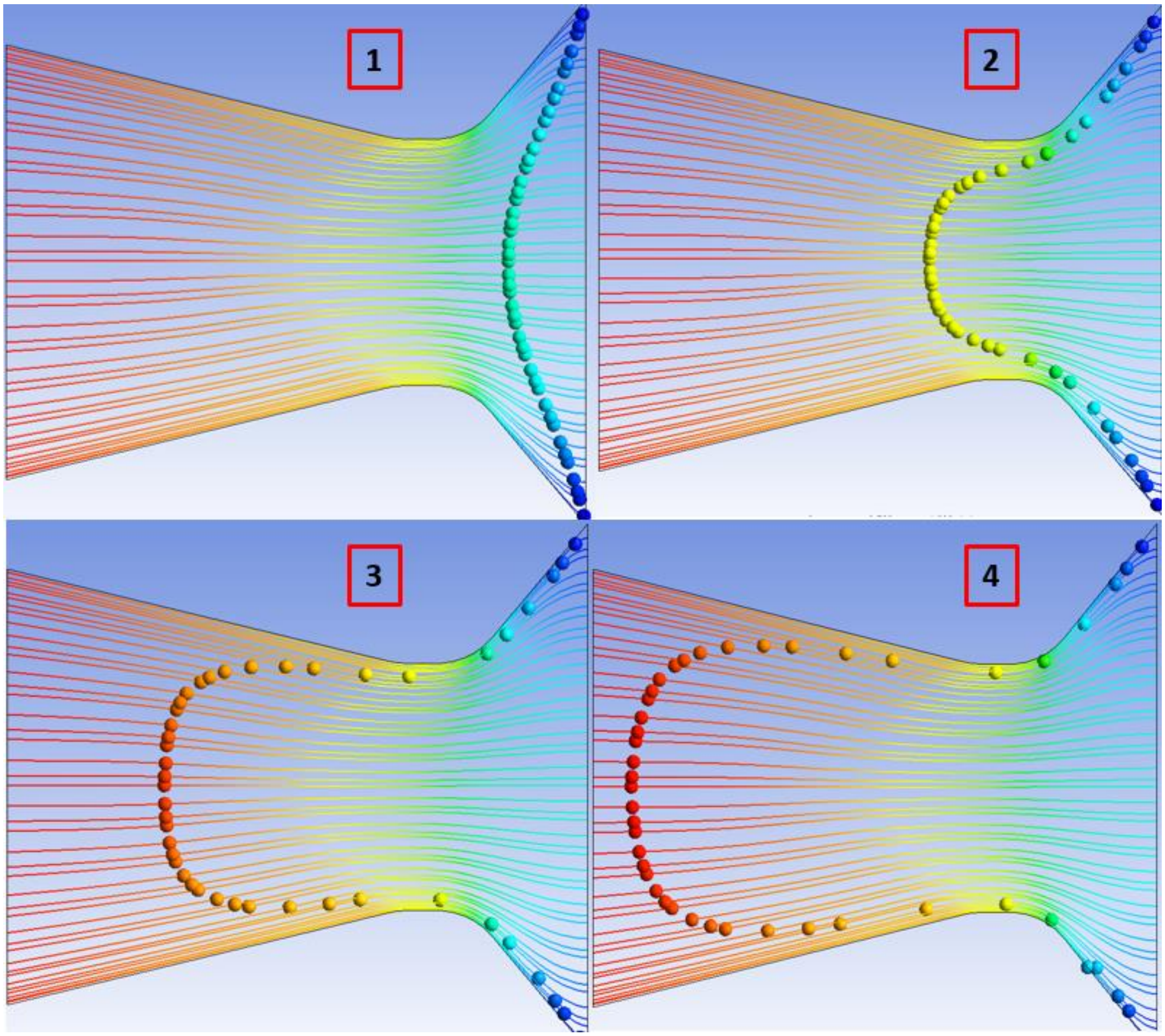


Figure 18 - Progression of Fluid Flow Through the Nozzle

Final Design

With the completion of the nozzle design, the motor design was completed. Modeling of the motor was done using Solidworks. **Figure 19** shows the 3D model of the completed motor, **Figure 20** shows the exploded view, and drawings of all components can be found in **Appendix 3**.

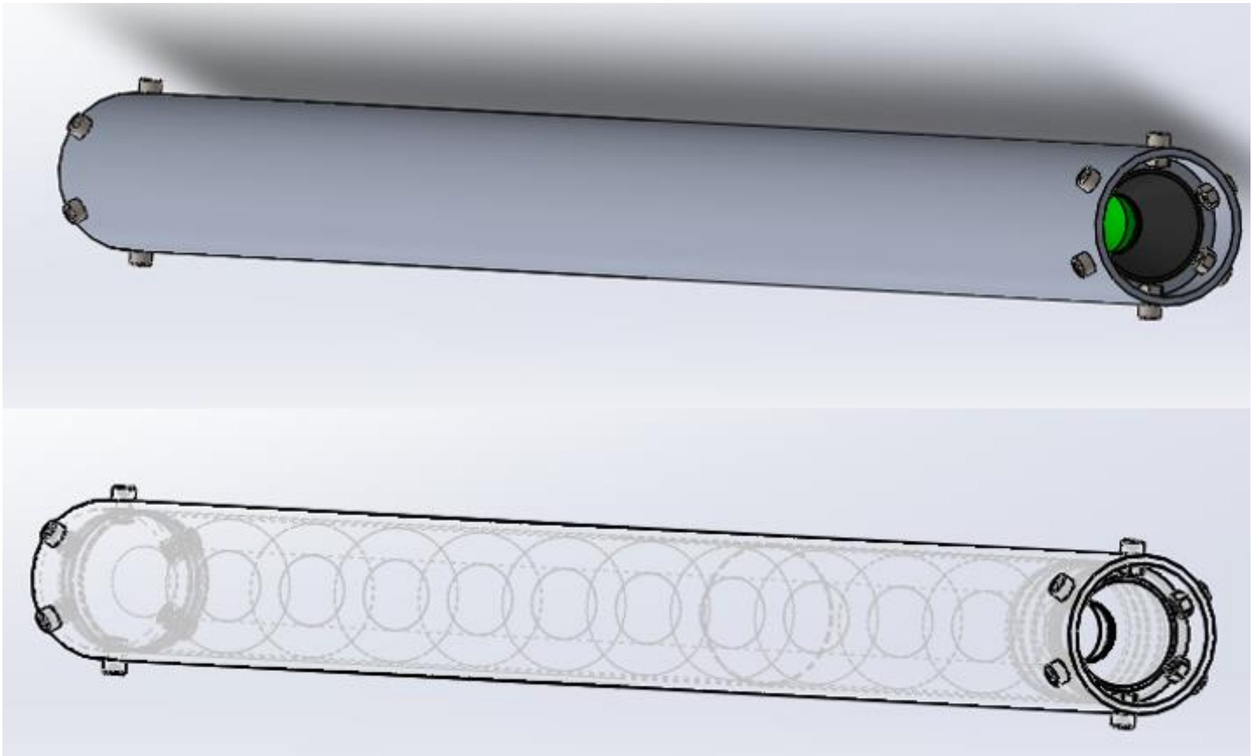


Figure 19 - 3D Model of Completed Motor

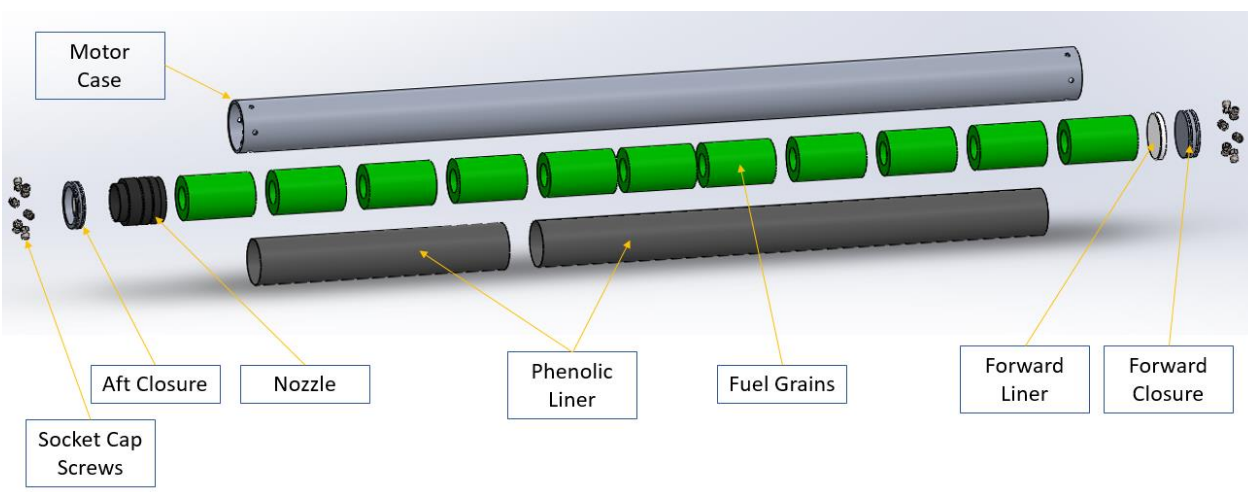


Figure 4 - Exploded View of Motor

Manufacture

All components are being machined in the Mechanical Engineering Machine shop in Auburn Science and Engineering Center. Once everything is machined, the motor will be assembled for use in Zoom.

Testing

IREC requires testing of all motors designed and built by students. A static test of the motor will be completed prior to competition. This test will assure the motor works properly and give data about the thrust the rocket produces. Having a real thrust curve will allow the team to better estimate the maximum altitude of the rocket.

References

[1] https://spaceflightsystems.grc.nasa.gov/education/rocket/specimp.html

[2] http://www.thrustcurve.org/motorstats.shtml

[3] https://spaceflightsystems.grc.nasa.gov/education/rocket/rktth1.html

[4] https://www.nakkarocketry.net/th_pix/grains4.gif

[5] https://www.rocketreviews.com/bates-grain.html

[6] http://www.makeitfrom.com/compare

[7] http://www.engineeringtoolbox.com/stress-thick-walled-tube-d_949.html

[8] https://www.boltdepot.com/fastener-information/materials-and-grades/bolt-grade-chart.aspx

[9] https://www.grc.nasa.gov/www/K-12/airplane/rktthsum.html

[10] http://www.engapplets.vt.edu/fluids/CDnozzle/cdinfo.html

[11] https://www.grc.nasa.gov/www/K-12/airplane/nozzled.html

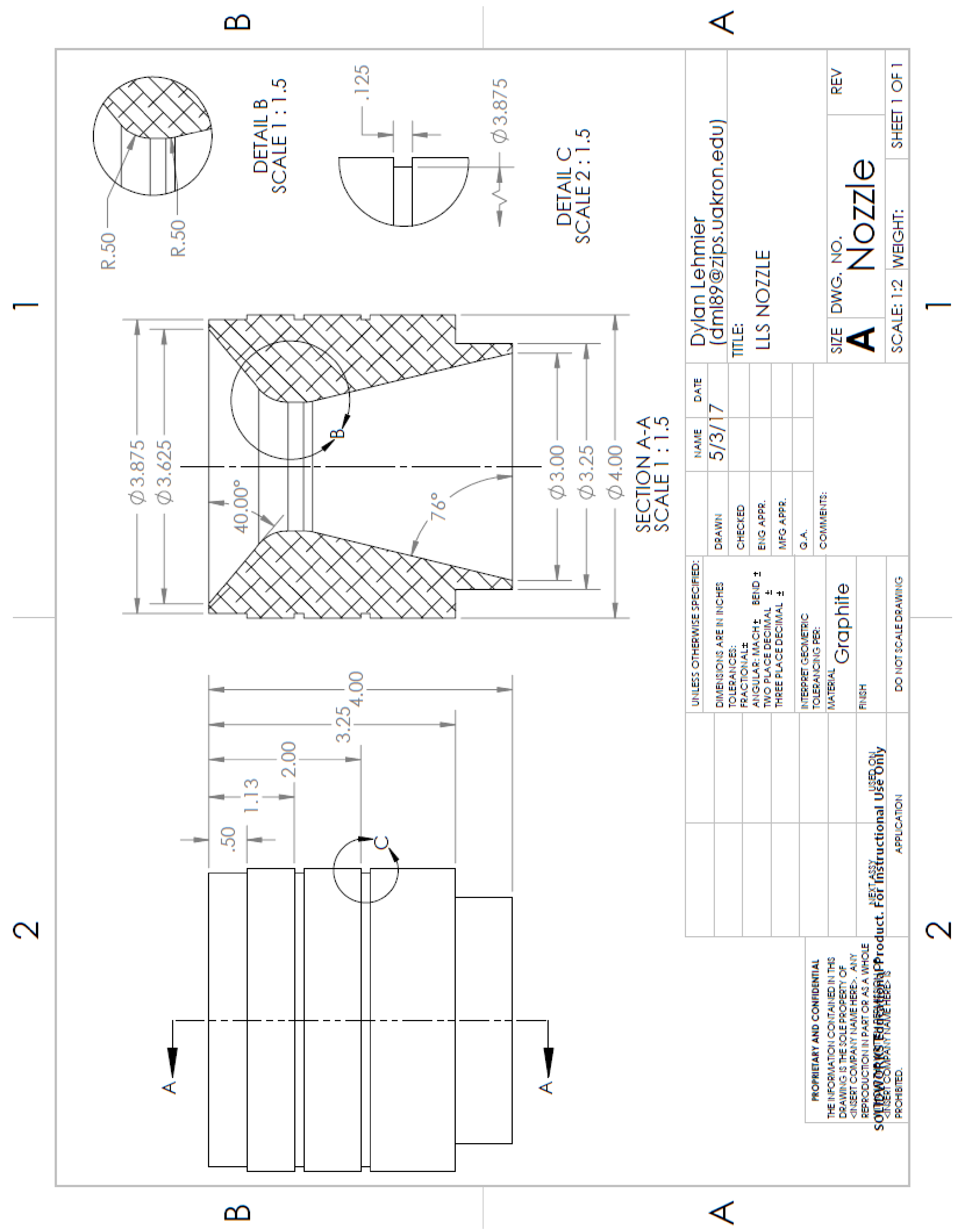
Appendix 1

```
clc
clear

%Thick walled pressure vessel stress calculations

%Known
OD = 4.5;       %inch
ID = 4.026;     %inch
r_o = OD/2;     %inch
r_i = ID/2;     %inch
r = r_i;        %inch (max stress occurs at r=r_i)
t = (OD-ID)/2;  %inch
Sy = 40000;     %psi (tensile yield strength for 6061 aluminum)

P_i = 744.2;    %psi (given from Burnsim)
P_o = 14.7;     %psi

%Calculating Axial, Circumferential, and Radial Stresses

sig_a = (P_i*r_i^2-P_o*r_o^2)/(r_o^2-r_i^2);
fprintf('The Axial Stress is %8.2f psi\n',sig_a)

sig_c = ((P_i*r_i^2-P_o*r_o^2)/(r_o^2-r_i^2)...
    -(r_i^2*r_o^2*(P_o-P_i))/(r^2*(r_o^2-r_i^2)));
fprintf('The Circumferential Stress is %8.2f psi\n',sig_c)

sig_r = ((P_i*r_i^2-P_o*r_o^2)/(r_o^2-r_i^2)...
    +(r_i^2*r_o^2*(P_o-P_i))/(r^2*(r_o^2-r_i^2)));
fprintf('The Radial Stress is %8.2f psi\n',sig_r)


%Calculate Failure using Distortion Energy Theory
for P_im = 1:10000
    sig_a = (P_im*r_i^2-P_o*r_o^2)/(r_o^2-r_i^2);

    sig_c = ((P_im*r_i^2-P_o*r_o^2)/(r_o^2-r_i^2)...
    -(r_i^2*r_o^2*(P_o-P_im))/(r^2*(r_o^2-r_i^2)));

    sig_r = ((P_im*r_i^2-P_o*r_o^2)/(r_o^2-r_i^2)...
    +(r_i^2*r_o^2*(P_o-P_im))/(r^2*(r_o^2-r_i^2)));

    sig_f = sqrt(((sig_c-sig_a)^2+(sig_a-sig_r)^2+(sig_r-sig_c)^2)/2);

    if sig_f >= Sy
       break

    end
    P_im;
end
fprintf('When the internal pressure reaches %4.1f psi, the stress exceeds the tensile
yield strength (%8.1f psi) and is likely to fail\n',P_im,Sy)


%Calculate the factor of safety of the motor casing

n=P_im/P_i;
fprintf('The factor of safety of the motor casing is %8.1f\n',n)
```

Appendix 2

```
clc
clear

% Calculations for using 6 screws

n=6;

% Known data for M10 x 15 class 12.9 screws

OD_b = 10; %mm
OD_b = OD_b/25.4; %convert to inches
A = n*OD_b/4*pi^2; %area in^2 for n screws
Sy_s = 160000; %psi (minimum yield strength)
Syy = 0.577*Sy_s; %psi (shear strength)

% Known data for 6061 T6 aluminum tube

OD_c = 4.5; %inch
ID_c = 4.026; %inch
t = (OD_c-ID_c)/2; %inch
A_id = ID_c/4*pi^2; %area of inside of case
Sy_a = 35000; %psi (minimum yield strength)

% Calculating the the safety factor for using 6 screws

sig_a = 2911.3; %psi (Axial stress found in motor case caluclations)
Fa = sig_a*A_id; %psi (Force seen by closures)
tao = Fa/A; %psi (shear stress)
SF_tao = Syy/tao; %Safety factor for using 6 screws
disp('Safety Factor for Shear of using 6 screws'); disp(SF_tao)

%bearing stress on bolts safety factor

A_b = n*OD_b*t;
sig_b = sig_a/A_b;
SF_b = Sy_s/sig_b;
disp('Safety Factor for bearing stress of using 6 screws'); disp(SF_b)

%bearing stress on motor hardware safety factor

SF_mh = Sy_a/sig_b;
disp('Safety Factor for bearing stress on hardware'); disp(SF_mh)
```

Appendix 3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Dylan Lehmier
(dml89@zips.uakron.edu)
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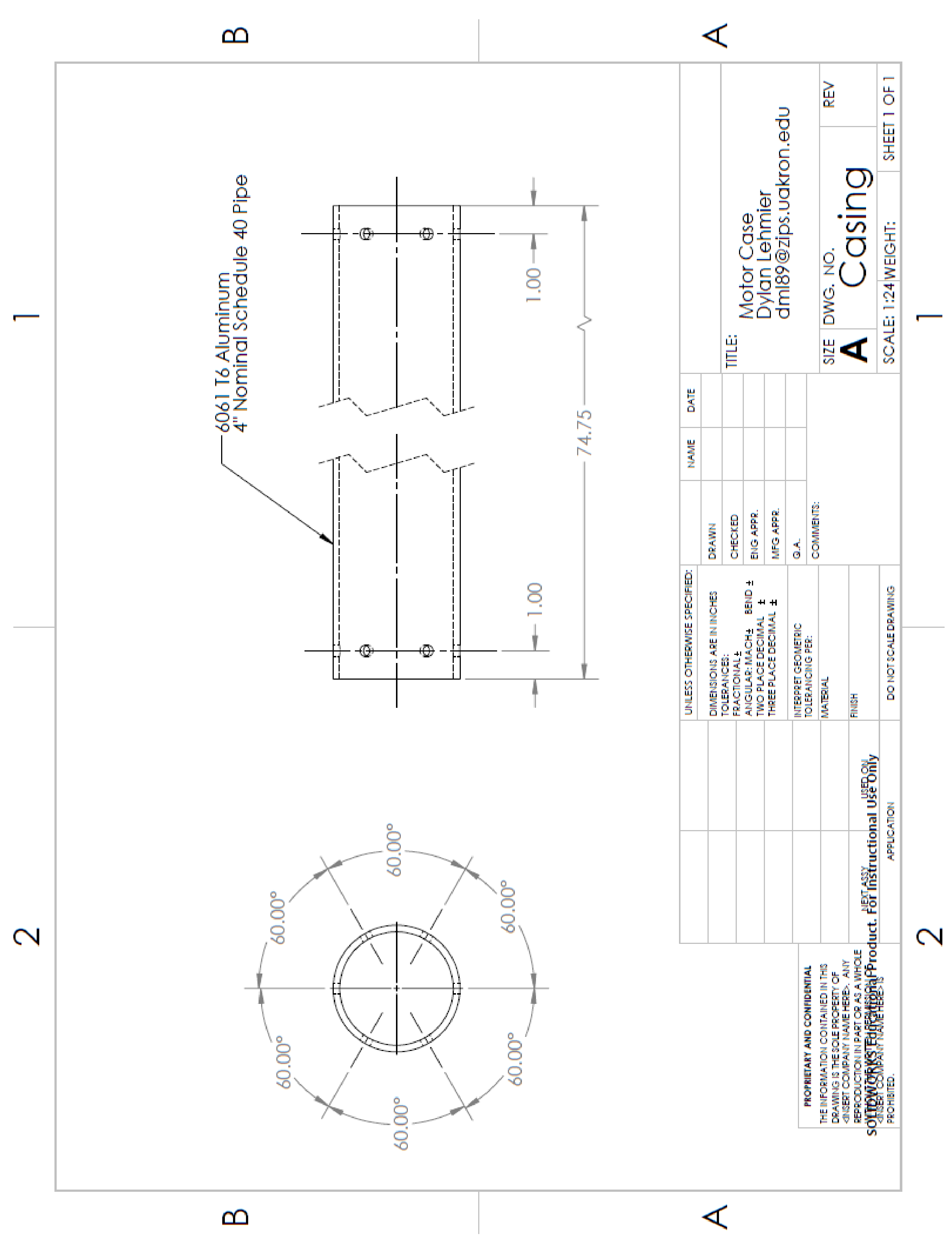
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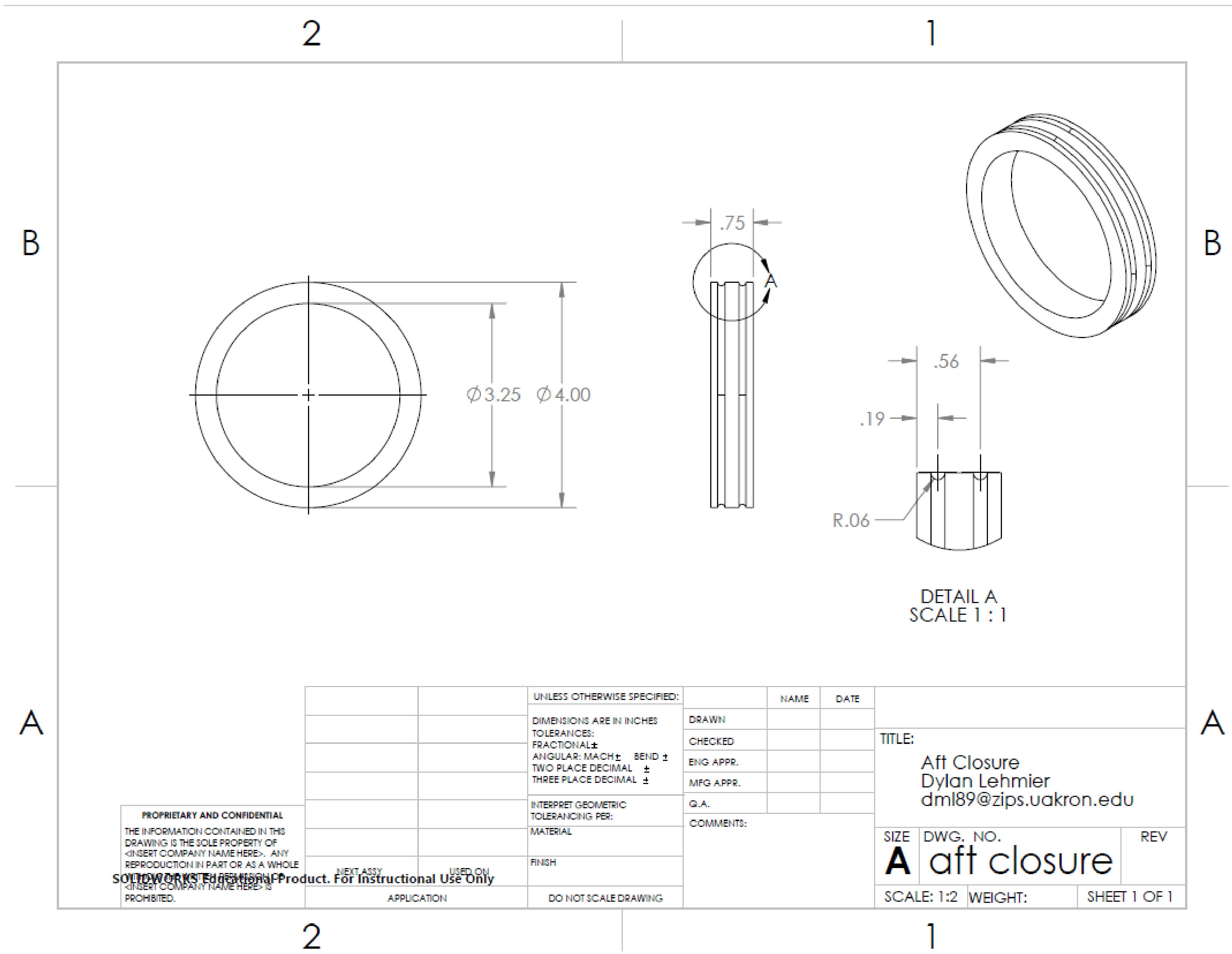

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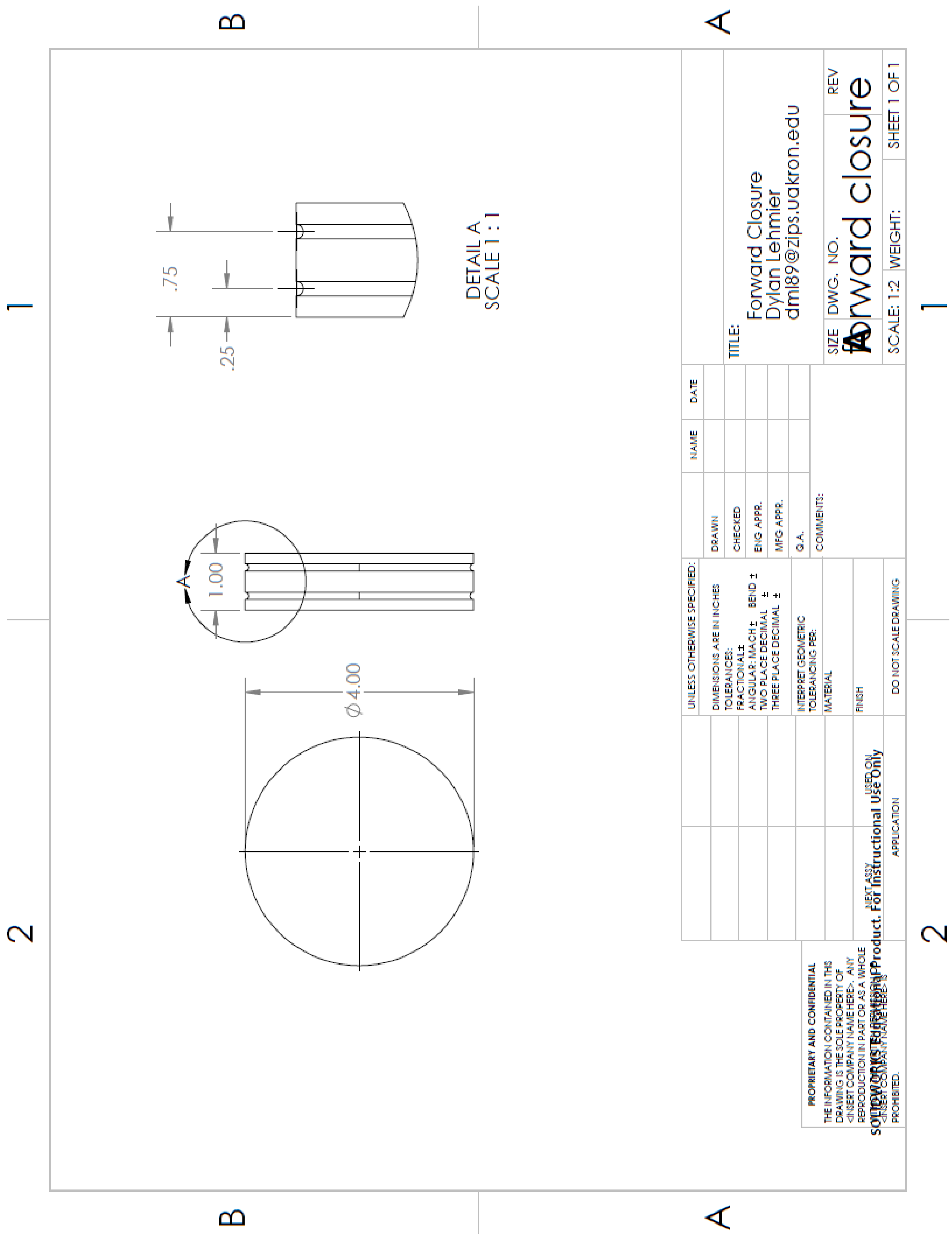

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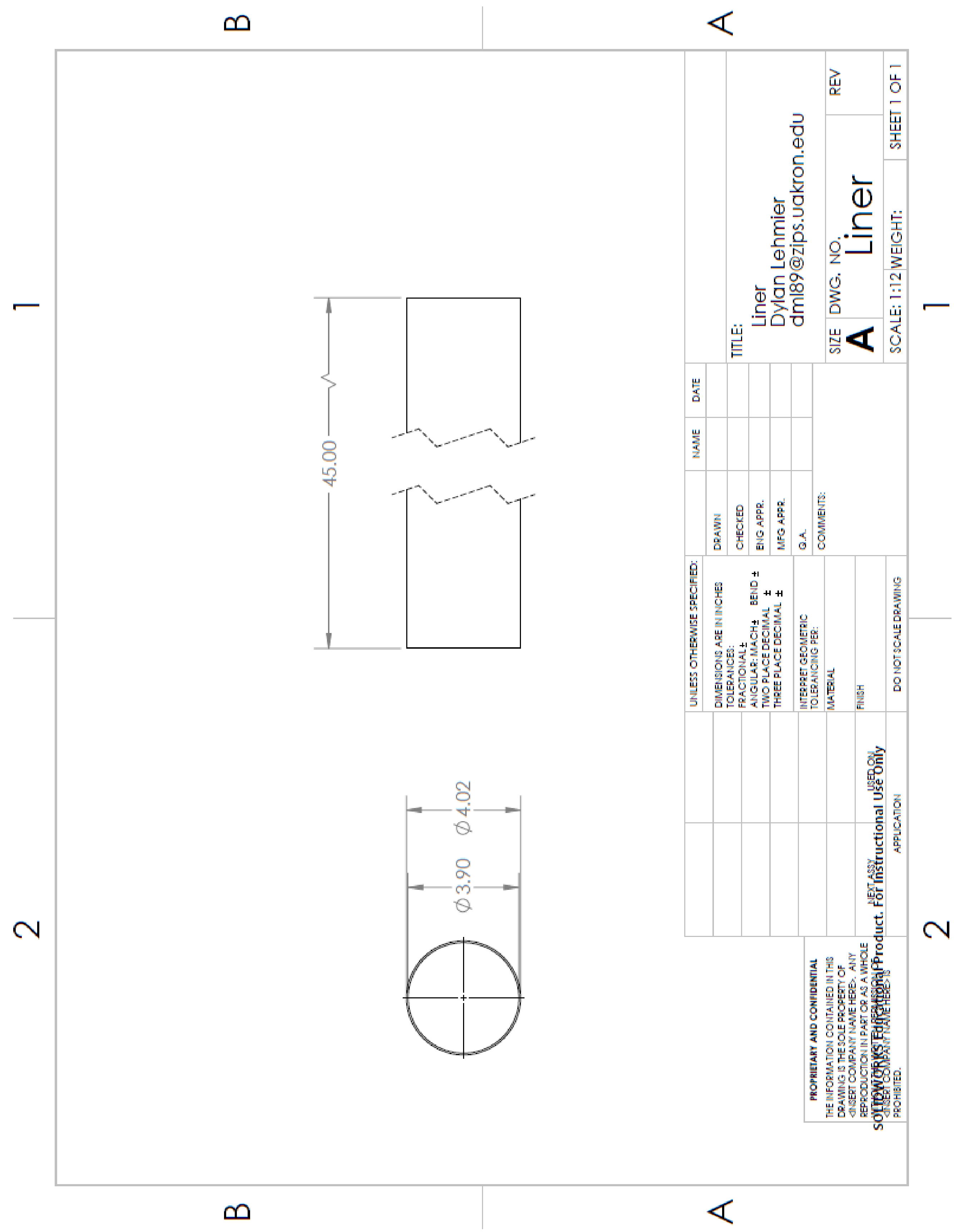
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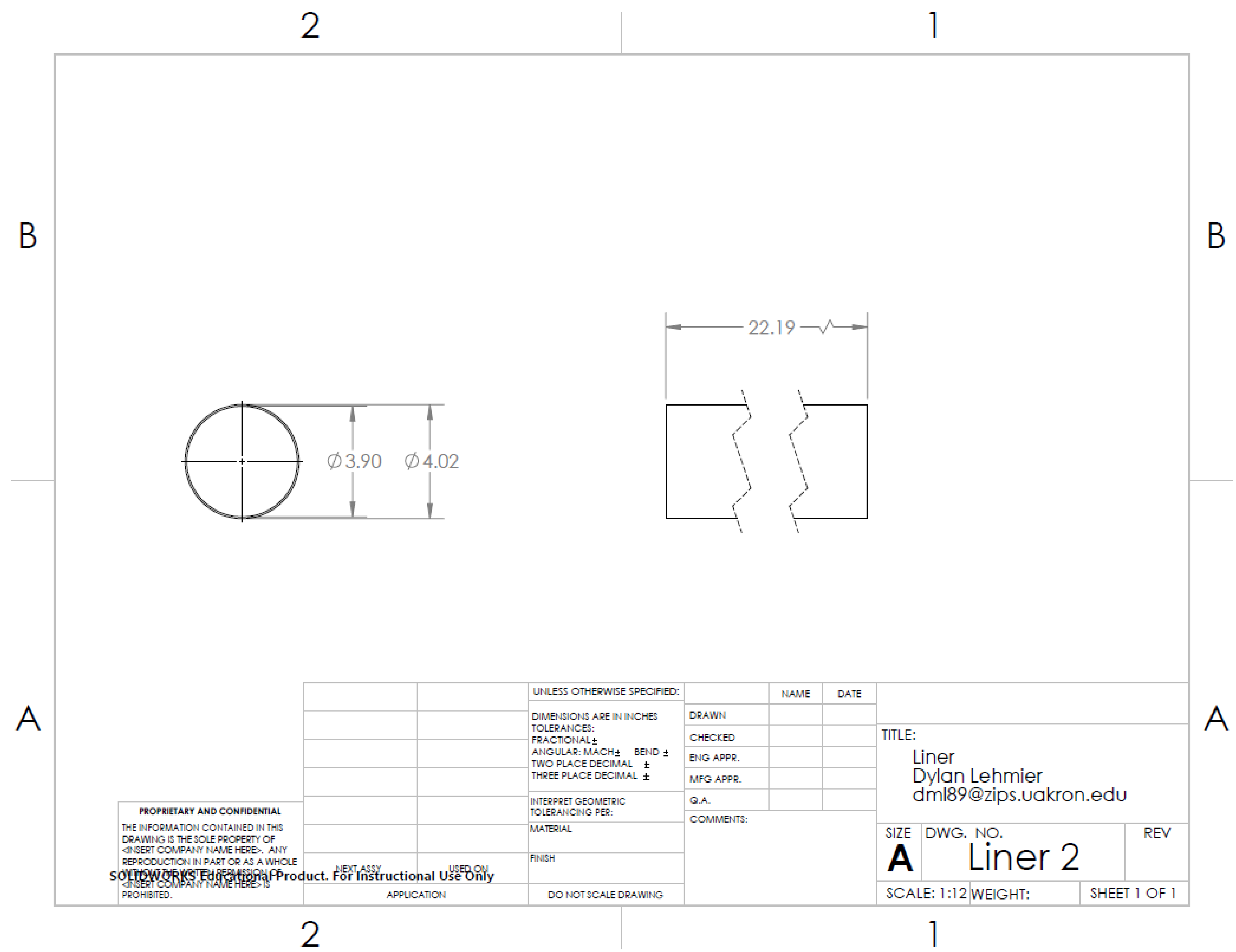

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